Exam 2, July 25, 2011.

Name _____

Work alone and use no books, notes, or calculators. Show your work with your answers on 8.5"×11" paper and staple the pages to the exam when you turn them in. Justify your conclusions.

Problem 1 (15 points) Find the derivatives **(a)** $y'(x)$ for $y(x) = \ln(x^3 + x^2)$, **(b)** $\dfrac{d}{dx}(\cos^2 x)$, and **(c)** $f'(2)$ for $f(x) = \tan^{-1} x$ (the inverse tangent function).

Problem 2 (10 points) Give an equation of the tangent line to $y = \sqrt{x^2 + 9}$ at $x = 4$.

Problem 3 (10 points) At a particular moment on a trip you are driving 60 miles per hour and your car is using gas at the rate of $\frac{1}{20}$ gallons per mile. At what rate are you using gasoline, measured in gallons per hour?

Problem 4 (10 points) A rectangular box with a square base and no top, as in Figure 1, is to be constructed to have a volume of $\frac{1}{2}$ cubic foot. Find the dimensions of the box that requires the least amount of material for the bottom and four sides.

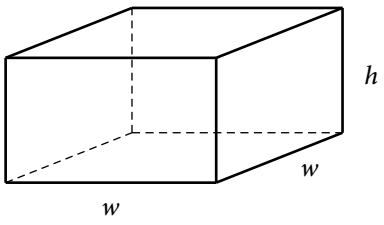

FIGURE 1

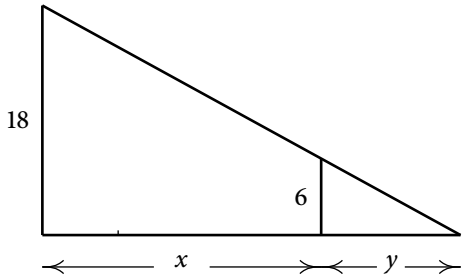

FIGURE 2

Problem 5 A six-foot-tall man is walking away from an eighteen-foot-high post with a lamp at its top (Figure 2). **(a)** (5 points) Find an equation relating his distance x from the post and the length y of his shadow. **(b)** (5 points) How fast is his shadow growing if he is walking two miles per hour?

Problem 6 (10 points) What is the rate of change of the width of a cube with respect to time at a moment when it is two inches wide and its volume is increasing 36 cubic inches per minute? ($V = w^3$)

Problem 7 **(a)** (3 points) Find the limits of $f(x) = x^3 - 3x^2 + 4$ as $x \to \pm\infty$. **(b)** (4 points) Find the open intervals on which f is increasing and decreasing and its local and global maxima and minima. **(c)** (4 points) Find the open intervals on which the graph of f is concave up and concave down and the inflection points of the graph. **(d)** (4 points) Draw the graph of f.

Problem 8 (10 points) Find **(a)** $\lim_{x\to\pi}\left(\dfrac{\sin x}{\sin(2x)}\right)$ and **(b)** $\lim_{x\to\infty}\left(\dfrac{\ln x}{x^2+1}\right)$.

Scores:

1	2	3	4	5	6	7	8	Total