1. Determine whether the following linear systems are consistent, and if so, find all the solutions.

(a).

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & x_3 & & & = & 3 \\ 2x_1 & + & 4x_2 & + & 2x_3 & + & x_4 & = & 9 \\ -x_1 & - & 2x_2 & & & + & 2x_4 & = & 7. \end{array}$$

(b).

$$\begin{array}{rcrcrcl} y_1 & - & y_2 & & & = & 1 \\ -2y_1 & + & 2y_2 & + & y_3 & = & -2 \\ 2y_1 & - & 2y_2 & + & y_3 & = & 0. \end{array}$$

$\frac{10}{15}$ (a) $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 3 \\ 2 & 4 & 2 & 1 & 9 \\ -1 & -2 & 0 & 2 & 7 \end{array}\right] \begin{array}{l} [2]-2\cdot[1] \\ [3]\cdot 2+[2] \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 5 & 16 \end{array}\right] [2]\Longleftrightarrow[3] \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 3 \\ 0 & 0 & 2 & 5 & 16 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$

$\boxed{\text{Consistent}}$ because there is a solution, i.e. no rows of $[0-0-0-0|X]$ $X \neq 0$

find solutions.

$\frac{10}{10}$ (b)

$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ -2 & 2 & 1 & -2 \\ 2 & -2 & 1 & 0 \end{array}\right] \begin{array}{l} [2]+[1]\cdot 2 \\ [3]+[2] \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -2 \end{array}\right] [3]-2[2]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right]$ $\boxed{\text{INCONSISTENT}}$ ✓

$\boxed{0 \neq -2}$

2. Let $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & -1 & -1 \\ 1 & 5 & 4 \end{bmatrix}$.

(a). Calculate the inverse of A.

(b). Use your answer to (a) to find a 3×3 matrix B solving the matrix equation

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

A

$$A^{-1} = \left[\begin{array}{ccc|ccc} -1 & 2 & 2 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 1 & 5 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ [2]+[1] \\ [3]-[2] \end{array} \Rightarrow \left[\begin{array}{ccc|ccc} -1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 6 & 5 & 0 & -1 & 1 \end{array}\right] \begin{array}{l} [1]-2\cdot[2] \\ \\ [3]-6\cdot[2] \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & -1 & -2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -6 & -7 & 1 \end{array}\right] \begin{array}{l} \cdot(-1) \\ [2]+[3] \\ \cdot(-1) \end{array} \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & -5 & -6 & 1 \\ 0 & 0 & 1 & 6 & 7 & -1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ -5 & -6 & 1 \\ 6 & 7 & -1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ -5 & -6 & 1 \\ 6 & 7 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 2 \\ 1 & -1 & -1 \\ 1 & 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B

$$B = A^{-1} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ -5 & -6 & 1 \\ 6 & 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot 1 + 2 + 0 & 1\cdot 1 + 2\cdot 0 + 0\cdot 0 & 1\cdot 0 + 2\cdot 0 + 0\cdot(-1) \\ (-5)\cdot 0 + (-6)\cdot 1 + 1\cdot 0 & (-5)\cdot 1 + (-6)\cdot 0 + 1\cdot 0 & (-5)\cdot 0 + (-6)\cdot 0 + 1\cdot(-1) \\ 6\cdot 0 + 7\cdot 1 + (-1)\cdot 0 & 6\cdot 1 + 7\cdot 0 + (-1)\cdot 6 & 6\cdot 0 + 7\cdot 0 + (-1)(-1) \end{bmatrix}$$

$$\boxed{B = \begin{bmatrix} 2 & 1 & 0 \\ -6 & -5 & -1 \\ 7 & 6 & 1 \end{bmatrix}}$$

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$$A\cdot B = \begin{bmatrix} -1 & 2 & 2 \\ 1 & -1 & -1 \\ 1 & 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 0 \\ -6 & -5 & -1 \\ 7 & 6 & 1 \end{bmatrix} = \begin{bmatrix} -2+14-12 & -1-10+12 & -2+2 \\ 2-1 & 1+5-6 & 1-1 \\ 2-30+28 & 1-25+24 & -5+4 \end{bmatrix} \overset{\checkmark}{=} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

3. $A = \begin{bmatrix} 2 & 4 & 1 & 0 & 4 \\ 1 & 2 & 1 & 1 & 6 \\ -1 & -2 & 1 & 1 & 4 \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent.

(There is no need to check this.)

(a). Give a definition of the column space of A.

(b). Find a basis for the column space of A.

(c). Give a definition for the null space of A.

(d). Find the null space of A.

(A) Col A = The columns of A where there are pivot Columns in B

+0

(B) Basis for Col A = $\begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ +5

These are linearly independent columns, equals ok for a Basis

(C) Null space of A is the ~~nontrivial~~ solution to the equation $A\vec{x} = \vec{0}$. where there are free variables, there is a null space.

+5

(d) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 - x_5 \\ x_2 \\ -2x_5 \\ -3x_5 \\ x_5 \end{bmatrix} \Rightarrow \boxed{x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ -2 \\ -3 \\ 1 \end{bmatrix}}$

basis? +6

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4. Let S be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by

$$S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{array}{cc} X_1 & X_2 \\ \end{array}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

and let T be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{array}{cc} X_1 & X_2 \\ \end{array}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

The linear transformation U is defined to be the composition:

$$U(\mathbf{x}) = T(S(\mathbf{x})). \qquad \text{for } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(a). Determine the matrix of the linear transformation U.

(b). Describe the linear transformation U geometrically.

$+0.$

$$T\left(S\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}\right) = T\left(\begin{array}{cc} Sx_1 & Sx_2 \\ \end{array}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\right) = \boxed{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}}$$

$$\left.\begin{array}{l} Tx_1 + Sx_1 \\ Tx_2 + Sx_2 \end{array}\right\} \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

(b) U is a shear transformation of a shear transformation

$+0.$

if I started of with a vector, I would have shifted the while rectangular image of it twice, ending up with some sort of parrallelogram.