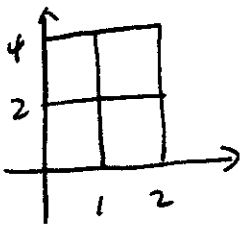


QUIZ 1 (TUESDAY)

Problem 1. Please answer the following:

- (1) (4 pts) Estimate the volume of the solid that lies below the surface $z = x + 2y^2$ and above the rectangle $R = [0, 2] \times [0, 4]$ using a Riemann sum with $m = n = 2$ and choosing the sample points to be the lower right corners.
- (2) (2 pts) Without computing the exact value of the integral, decide whether the estimate you are getting is smaller or larger than the actual value of the integral. Justify your reasoning.
- (3) (4 pts) Using the estimate from part 1, find the approximate value of the average of the function on the given rectangle.

(1) $f(x, y) = x + 2y^2$



$$\begin{aligned}
 (1, 2) \quad & V \approx 2 (f(1, 2) + f(2, 2) + f(1, 0) + f(2, 0)) \\
 (2, 2) \quad & = 2 (9 + 10 + 1 + 2) \\
 (1, 0) \quad & \\
 (2, 0) \quad & = 2 \cdot 22 = \boxed{44}
 \end{aligned}$$

2) Note that f increases as x increases and as y increases. However, the rate of increase in the y -direction is faster than that in the x -direction. By picking the lower right corners, we see that we are "overestimating in the x -direction" but "underestimating in the y -direction". Hence, our estimate should be smaller than the actual value since the decrease in y should be greater than the increase in x . Indeed:

$$\int_0^4 \int_0^2 (x + 2y^2) dx dy = \int_0^4 2 + 4y^2 dy = 2y + \frac{4y^3}{3} \Big|_0^4 = \frac{280}{3} > 44.$$

3) $\frac{44}{8} = \frac{11}{2}$.

QUIZ 1 (THURSDAY SECTIONS)

Problem 1. (10 pts) Find the volume of the solid enclosed by the paraboloid $z = 2 + x^2 + (y - 2)^2$ and the planes $z = 1$, $x = 1$, $x = -1$, $y = 0$ and $y = 4$.

$$\begin{aligned} V &= \int_{-1}^1 \int_0^4 1 + x^2 + (y - 2)^2 \, dy \, dx \\ &= \int_{-1}^1 4 + 4x^2 + \frac{(y-2)^2}{3} \Big|_0^4 \, dx \\ &= \int_{-1}^1 4 + 4x^2 + \frac{16}{3} \, dx \\ &= \frac{56}{3} + \frac{4x^3}{3} \Big|_{-1}^1 = \frac{56}{3} + \frac{8}{3} = \boxed{\frac{64}{3}} \end{aligned}$$