

**210 SEQUENCE COMPREHENSIVE EXAM - PART I (Morning)**

This exam is in two parts. The exam contains 5 questions and you are asked to answer all of them. Please be aware that you may be penalized for introducing irrelevant material.

Please note:

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- Use your signup number on each blue book, do not write your name
- Please write "Macro Comp" and the date on the cover of each blue book
- Answer each question in a separate blue book, write the question number on the outside of each blue book
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- The total length of the exam is 6 hours. The exam is in two parts, from 10am to 1pm, and 3pm to 6pm. At 1pm, you must turn in your exam and blue books to Susie Madsen in the Economics Academic Office. You may pick up the second part of the exam when the break ends at 3pm. If you opt to take the exam in its entirety with no break, you must turn in your blue books by 4pm.

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## Question 1

Consider the following two-capital goods economy.

(\*) The production technology is described

$$Y_t = Z_t K_t^{\alpha_k} S_t^{\alpha_s} N_t^{1-\alpha_k-\alpha_s}$$

where

$K_t$  = Physical Capital

$S_t$  = Structures

$N_t$  = Labor

$Z_t$  = Total Factor Productivity

Assume there is a measure one of firms operating this technology.

(\*) The household maximizes

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

(\*) The resource constraint in this economy is given by

$$Y_t = C_t + \frac{I_t^K}{Q_t^K} + \frac{I_t^S}{Q_t^S}$$

where  $I_t^K$  and  $I_t^S$  denote investment in the two types of capital.  $Q_t^K$  and  $Q_t^S$  denote investment specific shocks. For simplicity assume that the two types of capital fully depreciate at the end of the period.

(\*) The law of motion for each of the stocks of capital is

$$\begin{aligned} K_{t+1} &= I_t^K \\ S_{t+1} &= I_t^S \end{aligned}$$

(\*) You may assume that the household does the investment decisions and that the firm maximizes a static decision. That is, first, consumers save by purchasing the two types of capital; in the following period they then rent capital services to firms.

(a) Assume that this is a "social planner" economy. Formulate the Dynamic programming problem that the social planner solves. Be clear in defining the state variables both those that are endogenous as well as the exogenous ones.

(b) Derive the first order conditions that the social planner solves. You may assume that the value function is increasing in each of its arguments, strictly concave, differentiable for interior capital values

(c) Assume now that this economy is decentralized such that the firms own the two types of capital and the household can save via a risk free bond. Formulate the dynamic programming problem of the firm. Using the problem of the household, express the interest rate with which the firm discounts future revenues as a function of the marginal utility of consumption and  $\beta$ .

3.1. Consider the following model of monetary policy regime switching:

$$m_t - p_t = -\beta(E_t p_{t+1} - p_t) + \varepsilon_t \quad (1)$$

$$m_t = \alpha(s_t)p_t \quad (2)$$

where

$\beta$  = a positive coefficient

$m_t$  = log of the money supply

$p_t$  = log of the price level

$\varepsilon_t$  = serially uncorrelated random variable with mean 0 and variance  $\sigma^2$

Equation (1) describes money demand and equation (2) is a policy rule.

The coefficient of the policy rule follows a Markov process with two states and transition probability  $q_{ij}$  of going from state  $i$  to state  $j$

$$\alpha(s_t) = \begin{cases} \alpha_1 & \text{for } s_t = 1 \\ \alpha_2 & \text{for } s_t = 2 \end{cases}$$

a) Write down an explicit dynamic utility maximization problem for a representative agent which implies the type of money demand function in equation (1). You do not need to derive the equation. What other simplifying assumptions must be made about income and the real interest rate to result in the money demand function assumed here?

b) First suppose that  $\alpha_1 = \alpha_2 = \alpha$  (in other words, there is no regime switching).

- Under what conditions on  $\alpha$  will there be a unique rational expectations solution for the price level? Explain.

- Find the rational expectations solution for the log of the price level in terms of the random variable  $\varepsilon_t$ .

- Find the variance of  $p_t$ .

c) Now consider the switching regime case.

- Find the rational expectations solution for the log of the price level in terms of the random variable  $\varepsilon_t$ .

- Do the coefficients of the solution in one regime depend on the policy coefficients in the other regime?

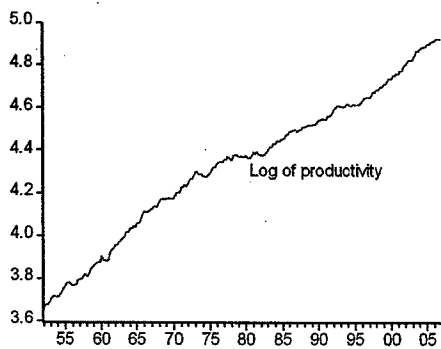
- Describe the path of the price level when there is a single shock to money demand which is temporary and unanticipated. Sketch the path in a simple graph, distinguishing between what happens in regime 1 and regime 2. Describe what happens to the expected rate of inflation and the nominal interest rate at the time of the shock and immediately afterwards.

3.2. State whether the following statements are true or false and briefly explain your answer using mathematics or graphs.

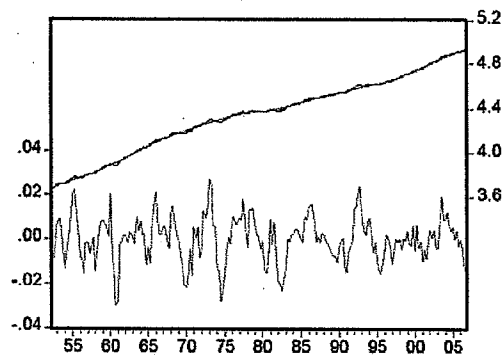
a) In the staggered contracts model, the persistence of shocks is longer than the length of the longest contract.

- b) It is impossible for money to Granger-cause output and for output to Granger-cause money at the same time in the same country
- c) The output-inflation variability frontier will shift toward the origin (toward lower inflation variance and output variance) if the sensitivity of inflation to output in the aggregate price adjustment equation (Phillips Curve) increases.
- d) The “consistent” (or “discretionary”) inflation rate depends on the weight put on output in the central bank’s loss function while the “optimal” (or “rule”) inflation rate is invariant to this weight.

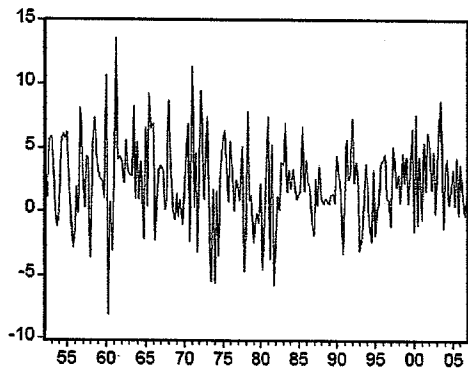
**3.3.** Five graphs are shown below. The first graph shows the log of labor productivity in the United States. The remaining four graphs show four different series for detrended log of productivity. Each graph corresponds to a different detrending method. Identify and define each of the four methods. Give the pros and cons of each method, including “Method 0 (no detrending)” when the purpose is to study the empirical relationship between monetary policy and economic fluctuations.



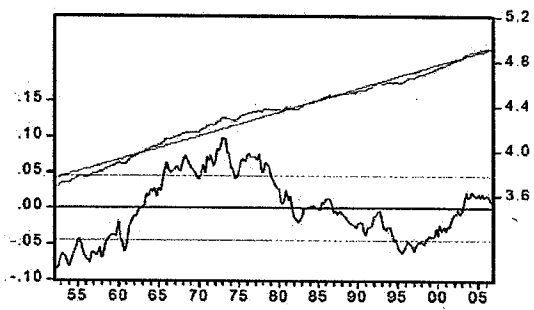
Method 1



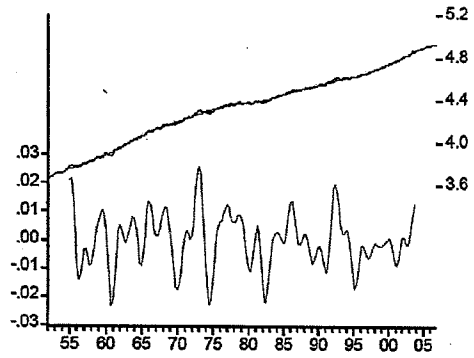
Method 2



Method 3



Method 4



**210 SEQUENCE COMPREHENSIVE EXAM - PART II (Afternoon)**

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## Question 4

Let a firm maximize the following Sequence Problem

$$\begin{aligned} \max \sum_{t=0}^{\infty} \rho^t [A_t F(K_t, N_t) - wN - I] \\ \text{s.t.} \\ K_{t+1} = I_t + (1 - \delta) K_t \end{aligned}$$

you may assume that  $F$  is constant returns to scale.  $w$  is the wage rate which the firm takes as given.  $A_t$  is total-factor productivity which the firm takes as given.

(a) Let the value function of the firm be denoted by  $V$ . Show that is the recursive formulation of the above sequence problem. Be clear in defining the state variables.

(b) Derive the FOC of the firm for capital. Denote by  $\eta$  the Lagrangian multiplier associated with the law of motion of capital

(c) Show that the value function of the firm is homogenous of degree one in capital. If you cannot show you may assume it. Hint: optimize out labor and derive the FOC.

(d) Define the end-of-period value of the firm as:

$$V^*(x', y) = E_0 \beta V(x', y''),$$

where  $x$  are endogenous states and  $y$  are exogenous states. That is, the true value function is  $V$ . I define above  $V^*$  as the value function that has an argument the next period endogenous state (which is known once the firm decides it today) and the current exogenous state. Use part (c) to show that you get that the end-of-period value of the firm implies

$$V^*(x', y) = x$$

What is  $x$  in this model? Explain the intuition. If you cannot prove argue what you think is the value of this firm

## 5. Under-Investment, Borrowing and Lack of Commitment in the Long Run

Consider a small open economy that receives foreign direct investment and borrows and lends a one-period bond. Technology is given by  $y = zf(k)$ , where  $k$  is foreign investment and  $z$  is an *iid* productivity shock. Utility of the domestic agent (or domestic government) is linear in consumption:  $U(c) = c$  for  $c \geq 0$ , and the domestic agent discounts the future with a factor  $\beta \in (0, 1)$ . The world interest rate is given by  $r$  and capital depreciates at a rate  $\delta$ .

An efficient allocation can be expressed as the optimal contract between foreigners and the domestic agent. In recursive form, the state variable is promised utility to the domestic agent  $v$ . The net payments to foreigners is given by:  $\mathbb{E}[zf(k) - (r + \delta)k - c(z)]$ , which includes the opportunity cost of capital. Denote by  $B(v)$  the value function of the foreigners given the promised value  $v$  to the domestic agent. Choice variables are capital  $k$ , consumption of the domestic agent in each state,  $c(z)$ , and continuation utility for the domestic agent in each state,  $\omega(z)$ . The Bellman equation for the problem is:

$$B(v) = \max_{c(z), \omega(z), k} \mathbb{E} \left[ zf(k) - (r + \delta)k - c(z) + \frac{1}{1+r} B(\omega(z)) \right]$$

subject to :

$$\begin{aligned} c(z) &\geq 0 \\ v &= \mathbb{E}[c(z) + \beta\omega(z)] \end{aligned}$$

Initially, we assume full commitment.

- (a) Solve for the optimal level of foreign direct investment. How does it depend on  $\beta$ ? Let the solution be denoted by  $k^*$ .

For the remainder of the question, suppose the domestic agent/government cannot commit to the allocation. If it deviates when the shock is  $z$  and capital is  $k$ , it gets  $zf(k)$  in that period (i.e. all the output) and then zero thereafter.

- (b) Write the participation constraints that need to be added to the constraint set of the above problem to ensure that the optimal allocation is equilibrium (the domestic agent does not deviate). You will need one per shock  $z$ .

**Ignore the non-negativity of consumption for what follows.** Assume that  $B(v)$  is concave and differentiable.

- (c) Write down the first order conditions of the problem without commitment. Show that either all participation constraints are binding or all are relaxed (that is, the multipliers on the participation constraints are all positive or all zero). What's the maximum value for the multiplier on the promised keeping constraint?
- (d) Let  $V^* = \mathbb{E}[zf(k^*)]$ . Show that the participation constraints do not bind for any  $v > V^*$ . Show that this implies  $k(v) = k^*$  and  $B'(v) = -1$  for  $v > V^*$ . Also, show that if  $v < V^*$ , then all participation constraints will be binding.

Assume that  $\beta(1+r) < 1$  and that  $B(v)$  is strictly concave for  $v < V^*$ .

- (e) From the first order conditions of the problem and show that  $w(z) = \tilde{v}$  where  $\tilde{v}$  solves:

$$B'(\tilde{v}) = -\beta(1+r)$$

which implies that if  $\beta(1+r) < 1$ ,  $\tilde{v} < V^*$ . If  $\tilde{v}$  unique?

- (f) Suppose that  $v = \tilde{v}$ . Use the envelope condition together with the first order conditions to argue that  $k(v)$  is given by the value  $\bar{k}$  that solves

$$\mathbb{E}[zf'(\bar{k})] = \frac{r+\delta}{\beta(1+r)}$$

How does  $\bar{k}$  vary with  $\beta$  now? Compare with (a).

- (g) Bring back the non-negativity constraint on consumption. When is  $\tilde{v}$  a steady state of the problem? *Hint: Show that all the participation constraints are binding at  $\tilde{v}$ , and use that to argue that  $\tilde{v} = \mathbb{E}[zf(\bar{k})]$ . Then, from a binding participation constraint, solve for consumption and find a condition for consumption to be always positive.*