PROBLEM 1.

The position of a simple harmonic motion as a function of time is given by $x = 2.0\cos(2\pi t/3 + \pi/6)$ where t is in seconds and x in meters. Find
(a) the period and frequency,
(b) the velocity and acceleration at $t = 2.0$ s,
(c) the total distance traveled between $t = 1.0$ s and $t = 2.0$ s.

(a) $\omega = 2\pi/3 \Rightarrow$

$$T = \frac{2\pi}{\omega} = 3\ s$$

$$f = 1/T = 1/3\ Hz$$

(b) $v = -\frac{4\pi}{3}\sin(2\pi t/3 + \pi/6)$

$$v(t=2) = +4\pi/3\ m/s = 4.19\ m/s$$

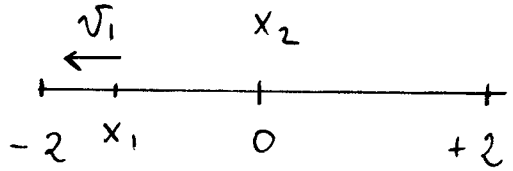
$$a = -\frac{8\pi^2}{9}\cos(2\pi t/3 + \pi/6)$$

$$a(t=2) = 0\ m/s^2$$

(c) $\Delta t = 1\ s < T\ (= 3\ s) \Rightarrow$ less than 1 cycle.

$x_1 = x(t=1) = -1.73\ m \quad v_1 = v(t=1) < 0$

$x_2 = x(t=2) = 0\ m$



$\Rightarrow$ $d = (2 + 0.27)\ m = 2.27\ m$

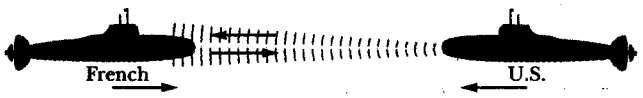
PROBLEM 2.

A French submarine and a U.S. submarine move head-on during maneuvers in motionless water in the North Atlantic (see figure). The French sub moves at 14 m/s and the U.S. sub at 20 m/s. The French sub sends out a sonar signal (sound wave in water) at 1000 Hz. Sonar waves travel at 1500 m/s.

(a) What is the signal's frequency as detected by the U.S. sub?

(b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?

(c) If the frequency detected by the French sub in the signal reflected back to it by the U.S. sub was 1030 Hz instead, what would be the U.S. sub speed?



(a)

$$f' = \left(\frac{v + v_D}{v - v_S}\right) f$$

with: $v = 1500$ m/s, $v_D = +20$ m/s, $v_S = +14$ m/s, $f = 1000$ Hz

$$\boxed{f' \simeq 1023 \text{ Hz}}$$

(b) same formula but with: $v_D = +14$ m/s, $v_S = +20$ m/s, $f = 1023$ Hz

$$\boxed{f' \simeq 1047 \text{ Hz}}$$

(c) $$f' = \frac{v + v_{US}}{v - v_F} f \quad , \quad f'' = \frac{v + v_F}{v - v_{US}} f'$$

Then $$f'' = \frac{v + v_F}{v - v_{US}} \cdot \frac{v + v_{US}}{v - v_F} f$$

with: $v = 1500$ m/s, $f'' = 1030$ Hz, $f = 1000$ Hz, $v_F = 14$ m/s

$$\Downarrow$$

$$\boxed{v_{US} = 8.2 \text{ m/s}}$$

PROBLEM 3.

A string is driven into wave motion at a frequency of 10.0 Hz. The amplitude of the motion is 10.0 cm and the wave travels at a speed of 10.0 m/s to the left (in the $-x$ direction). Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$. Determine

(a) the angular frequency, and

(b) angular wave number for this wave.

(c) Write an expression for the wave function.

Calculate

(d) the maximum transverse speed and

(e) the maximum transverse acceleration of an element of the string.

Include units in your answers.

SOLUTION 3.

(a) $\omega = 2\pi f = (2\pi \text{ rad/cycle})(10.0 \text{ cycle/s}) = \boxed{62.8 \text{ rad/s}}$

(b) $k = \omega/v_x = (62.8 \text{ rad/s})/(10.0 \text{ m/s}) = \boxed{0.628 \text{ rad/m}}$

(c) $y(x,t) = A\sin(kx + \omega t + \phi) = \boxed{(10.0 \text{ cm})\sin\big[(0.628 \text{ rad/m})x + (62.8 \text{ rad/s})t\big]}$

The sign in front of ω is positive since the wave is travelling in the $-x$ direction.

The phase constant is ϕ is taken to be zero because $y(0,0) = A\sin(\phi) = 0$.

Given the units used, y is in centimeters, x is in meters, and t is in seconds.

(d) $v_{y,\max} = \omega A = (62.8 \text{ rad/s})(10.0 \text{ cm}) = \boxed{628 \text{ cm/s}}$

(e) $a_{y,\max} = \omega^2 A = (62.8 \text{ rad/s})^2(10.0 \text{ cm}) = 3.95 \times 10^4 \text{ cm/s}^2 = \boxed{395 \text{ m/s}^2}$