Chapter 1. Limits and Continuity

1.1. Rates of Change and Limits

Definition. The *average rate of change* of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}$$

where $h = x_2 - x_1$.

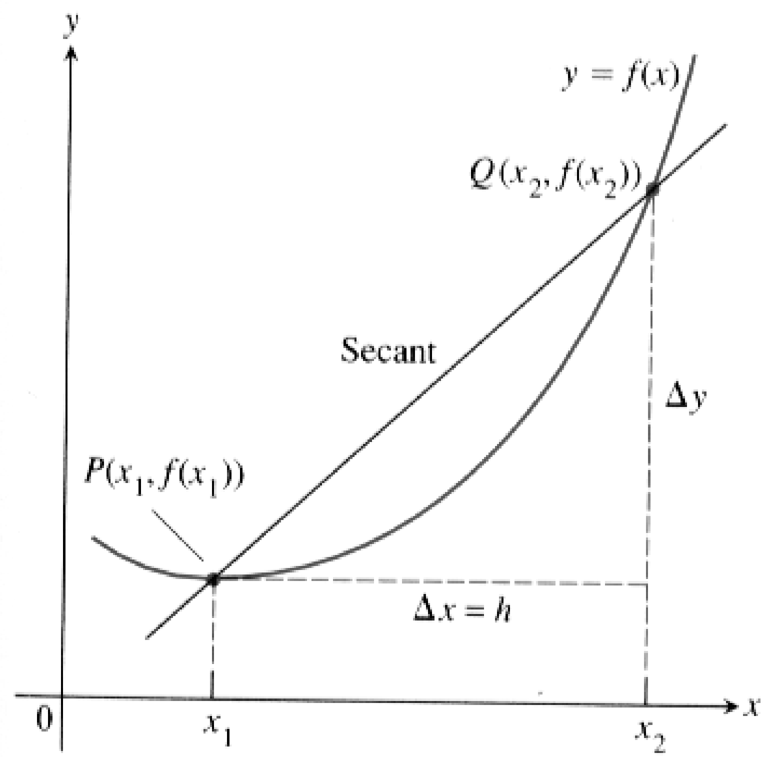

Figure 1.1.1, page 87

Example. Page 95 number 4.

Definition. Informal Definition of Limit.

Let $f(x)$ be defined on an open interval about x_0, **except possibly at x_0 itself**. If $f(x)$ gets arbitrarily close to L for all x sufficiently close to x_0, we say that f *approaches the limit* L *as* x *approaches* x_0, and we write

$$\lim_{x \to x_0} f(x) = L.$$

Note. The above definition is **informal** (that is, it is not mathematically rigorous) since the terms "arbitrarily close" and "sufficiently close" are not defined.

Example. Page 96 number 10.

Example. Page 97 number 20.

Definition. Formal Definition of Limit

Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say that $f(x)$ approaches the *limit* L *as* x *approaches* x_0 and write

$$\lim_{x \to x_0} f(x) = L,$$

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x,

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

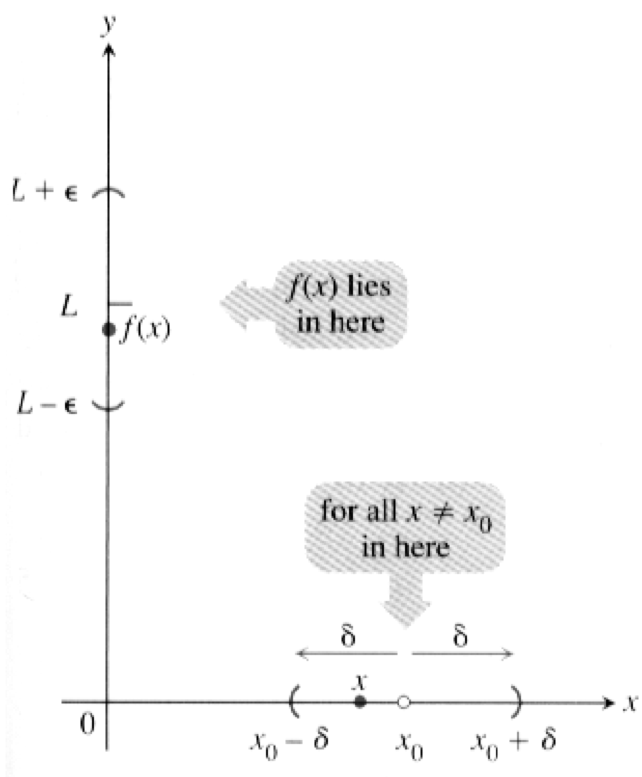

Figure 1.1.11, page 93

Example. Prove for $f(x) = mx + b$, $m \neq 0$, that $\lim_{x \to a} f(x) = f(a)$.

Example. Page 97 number 30.

Example. Page 97 number 34.

Example. Page 98 number 42.